Dynamics with infinitely many time derivatives in Friedmann-Robertson-Walker background and rolling tachyons

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ABSTRACT: Dynamics with infinitely many time derivatives has place in string field theory and have been profoundly investigated there. Recently there has been considerable interest in theories with infinitely many derivatives in the cosmological context in view of new features which these theories might accommodate owing to nonlocal interaction. In present work we continue investigation of such models, as a concrete example we study the dynamics of unstable D-brane in the open string theory in the Friedmann-Robertson-Walker background. We construct numerical solutions describing dynamical interpolation between the perturbative and non-perturbative vacua. The obtained solutions have several interesting properties and might be of interest from the cosmological points of view.

KEYWORDS: Cosmology of Theories beyond the SM, Tachyon Condensation, String Field Theory.

Contents

1. Introduction

Consideration of fundamental theories such as M/String Theory in the cosmological context continues to attract attention in the literature. One of the interesting questions is the role of tachyon in String Theory and Cosmology. The great progress in our understanding of tachyon condensation was made in the past decade [1–3], but a lot of interesting issues are still open. Among the most important ones is a better understanding of the dynamics of tachyon condensation process.

In this context many works have been devoted to the study of time-dependent solutions. Probably one of the most fascinating frameworks for this is Open String Field Theory (OSFT) [4] which reattracted a lot of attention after a recent work [5]. Despite the recent renewal of interest to OSFT no smooth solutions interpolating between two inequivalent vacua even at the lowest level truncation order [6, 7] were found. One of exceptional features of the level truncation approach is that corresponding action contains infinitely many time derivatives, i.e. it is non-local. Resulting models have a rich set of properties that might be essential for the development of stringy cosmology. Substantial investigation

of this type of models was performed [8, 9, 18−33, 35−37, 43, 42] after seminal paper [8] where many useful properties were revealed, among which the presence of rolling solution with widely increasing oscillations. Further research of time-dependent rolling solutions was also performed in particular in [10]. Recently several time dependent rolling tachyon solutions in OSFT were found [12, 13], which confirmed puzzling behavior of the solutions found earlier [8, 10, 11]. In these works investigations were performed in usual space-time coordinates. At the same time is it known that in the light-cone gauge the theory becomes local in light-cone time [14]. In fact the question of the identity of light-like and time-like cases is under investigation. In the light-like case (with dilatonic damping), the gradient flow forces tachyon to asymptote to the true vacuum at late times [15]. In the time-like case, rolling configuration between two non-equivalent vacua is forbidden by energy conservation law in Minkowski space time if one considers only tachyon excitation in the level truncation approximation [8, 16]. Investigations in the last case though did not take into account any effects of gravity, this makes such investigations inconsistent from cosmological point of view.

It turns out that if we consider the same action coupled to the gravity in Friedmann-Robertson-Walker (FRW) metric the situation changes: there appears a tachyon solution which tends to the true vacuum at late times [16]. It is interesting to note that because dilaton appears from the same string sector as graviton, inclusion of dilaton into the tachyon action can qualitatively reproduce behavior of the tachyon in curved space. Thus obtained solution is in accordance with results obtained in light-like and time-like cases at least at the lowest level approximation.

In the present work we consider scalar field dynamics with infinitely many time derivatives minimally coupled to the Minskowski and Friedmann-Robertson-Walker gravitational backgrounds.

The structure of the work is the following. In the first section we will give a brief introduction and physical motivation. In the second section the model which appears from Open String Field Theory will be presented and problem of existence of solutions interpolating between two inequivalent vacua will be discussed. In the third section we will consider the model minimally coupled to gravity and demonstrate an intriguing difference compared to the case without gravity: the existence of desired solution. Numerical techniques will be described in section 4. Results of numerical calculations will be presented in section 5. Finally we will summarize main results.

2. The model

The action of bosonic cubic string field theory has the form

$$S = -\frac{1}{g_0^2} \int \left(\frac{1}{2} \Phi \cdot Q_B \Phi + \frac{1}{3} \Phi \cdot (\Phi * \Phi) \right), \tag{2.1}$$

where g_0 is the open string coupling constant, Q_B is BRST operator, $*$ is noncommutative product and Φ is the open string field containing component fields which correspond to all the states in string Fock space.

Considering only tachyon field $\phi(x)$ at the level (0,0) the action (2.2) becomes

$$S = \frac{1}{g_0^2} \int d^{26}x \left[\frac{\alpha'}{2}\phi(x)\Box\phi(x) + \frac{1}{2}\phi^2(x) - \frac{1}{3}K^3\Phi^3(x) - \Lambda \right], \qquad (2.2)$$

where α' is the Regge slope, $K = \frac{3\sqrt{3}}{4}$, ϕ is a scalar field, $\Phi = e^{k\Box}\phi$, $k = \alpha'\ln K$, $\Box = \frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu$ and $\Lambda = \frac{1}{6}K^{-6}$ was added to the potential to set the local minimum of the potential to zero according Sen's conjecture [38]. In what follows we will work in units where $g_0 = 1$.

The action (2.2) leads to equation of motion

$$(\alpha'\Box + 1)e^{-2k\Box}\Phi = K^3\Phi^2. \qquad (2.3)$$

The Stress Tensor for our system is[1]

$$T_{\alpha\beta}(x) = -g_{\alpha\beta}\left(\frac{1}{2}\phi^2 - \frac{\alpha'}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{3}K^3\Phi^3 - \Lambda \right) - \alpha'\partial_\alpha\phi\partial_\beta\phi$$

$$-g_{\alpha\beta}\, k \int_0^1 d\rho \left[(e^{k\rho\Box}K^3\Phi^2)(\Box e^{-k\rho\Box}\Phi) + (\partial_\mu e^{k\rho\Box}K^3\Phi^2)(\partial^\mu e^{-k\rho\Box}\Phi) \right] \quad (2.4)$$

$$+2k \int_0^1 d\rho \, (\partial_\alpha e^{k\rho\Box}K^3\Phi^2)(\partial_\beta e^{-k\rho\Box}\Phi).$$

On spatially homogeneous configurations the energy is defined as $\mathcal{E}(t) = T^{00}$ and pressure as $\mathcal{P}(t)_i = -T_i^i$ (no summation) and for our system

$$\mathcal{E} = \mathcal{E}_k + \mathcal{E}_p + \Lambda + \mathcal{E}_{nl1} + \mathcal{E}_{nl2}, \qquad \mathcal{P} = \mathcal{E}_k - \mathcal{E}_p - \Lambda - \mathcal{E}_{nl1} + \mathcal{E}_{nl2} \qquad (2.5)$$

where

$$\mathcal{E}_k = \frac{\alpha'}{2}(\partial\phi)^2, \qquad \mathcal{E}_p = -\frac{1}{2}\phi^2 + \frac{K^3}{3}\Phi^3$$

$$\mathcal{E}_{nl1} = k \int_0^1 d\rho \left(e^{k\rho\Box}K^3\Phi^2 \right) \left(-\Box e^{-k\rho\Box}\Phi \right),$$

$$\mathcal{E}_{nl2} = -k \int_0^1 d\rho \left(\partial e^{k\rho\Box}K^3\Phi^2 \right) \left(\partial e^{-k\rho\Box}\Phi \right). \qquad (2.6)$$

where $\Box = -\partial^2$ and ∂ denotes time derivative.

The symbol $e^{\rho\partial^2}\varphi$ defined as[2]

$$e^{\rho\partial^2}\Phi(t) = C_\rho[\Phi](t) \qquad (2.7)$$

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[1]Note that here and below integration over ρ is understood as limit of the following regularization

$$\int_0^1 d\rho f(\rho) = \lim_{\epsilon_1 \to +0} \lim_{\epsilon_2 \to +0} \int_{\epsilon_1}^{1-\epsilon_2} d\rho f(\rho).$$

[2]It is easier to use integral representation for the operator $e^{\rho\partial^2}$ while considering Minkowski background, in the FRW case though it becomes impossible to generalize such an approach and we define the operator $e^{\rho\partial^2}$ in terms of solution of the boundary value problem for diffusion equation, see [31] for the details.

where

$$C_\rho[\Phi](t) = \frac{1}{\sqrt{4\pi\rho}} \int_{-\infty}^{+\infty} e^{-\frac{(t-t')^2}{4\rho}} \Phi(t')dt'.$$

Nonlocal terms $\mathcal{E}_{nl_1}$ and $\mathcal{E}_{nl_2}$ contain $e^{-k\rho\partial^2}$ which as it can be easily seen might lead to the growing kernel in the integral representation of the non-local operator, that is why we will try to avoid calculation of $e^{-k\rho\partial^2}$ and will use the following representation for nonlocal energy terms $\mathcal{E}_{nl1}$ and $\mathcal{E}_{nl2}$ which are valid on the equation of motion for the scalar field

$$\mathcal{E}_{nl1} = k \int_0^1 d\rho \left((-\alpha'\partial^2 + 1)e^{(2-\rho)k\partial^2}\Phi \right) \left(\partial^2 e^{k\rho\partial^2}\Phi \right),$$

$$\mathcal{E}_{nl2} = -k \int_0^1 d\rho \left(\partial(-\alpha'\partial^2 + 1)e^{(2-\rho)k\partial^2}\Phi \right) \left(\partial e^{k\rho\partial^2}\Phi \right). \qquad (2.8)$$

2.1 Energy conservation

Energy conservation laws always have the deep physical sense. Note that because of presence of infinitely many time derivatives we need to prove energy conservation explicitly. Similar investigation was performed in [8], although in that work representation of pseudo-differential operator via the summation over infinite series expansion was used with which one always needs to be very careful with regard to convergence issues. The approach used here is advantageous from the point of view of numerical calculations, because in order to define action of the exponential operator we need to do only one well defined integration.

Taking into account that models with different types of potentials are currently under the consideration in the literature [29, 31, 43] we will show the energy conservation for arbitrary potential.

Claim 1. [3] *The Energy*

$$\mathcal{E} = \frac{\alpha'}{2}(\partial\phi)^2 - \frac{1}{2}\phi^2 + V(\Phi) + \Lambda + k \int_0^1 d\rho \, ((-\alpha'\partial^2 + 1)e^{(2-\rho)k\partial^2}\Phi) \overset{\leftrightarrow}{\partial} (\partial e^{k\rho\partial^2}\Phi),$$

is conserved on the solutions of equation of motion

$$(-\alpha'\partial^2 + 1)e^{2k\partial^2}\Phi = \frac{\partial V(\Phi)}{\partial\Phi}$$

of the corresponding action

$$S = \frac{1}{g_0^2} \int d^{26}x \left[\frac{\alpha'}{2}\phi(x)\Box\phi(x) + \frac{1}{2}\phi^2(x) - V(\Phi) - \Lambda \right],$$

where $V(\Phi)$ is any polynomial potential and $A \overset{\leftrightarrow}{\partial} B = A\partial B - B\partial A$.

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[3]For simplicity we will use symbolic notation for nonlocal operator e^{∂^2} keeping in mind that it is in fact defined by (2.7), also integration over ρ must be understood as limit of the corresponding regularization as indicated earlier.

Proof. We compute

$$\frac{d\mathcal{E}(t)}{dt} = \alpha'\partial\phi\partial^2\phi - \phi\partial\phi + \frac{\partial V(\Phi)}{\partial\Phi}\partial\Phi + k\int_0^1 d\rho((-\alpha'\partial^2 + 1)e^{(2-\rho)k\partial^2}\Phi)\overleftrightarrow{\partial^2}(\partial e^{k\rho\partial^2}\Phi)$$

Using following identity (for details on its derivation see appendix)

$$\int\limits_0^1 d\rho(e^{\rho\partial^2}\varphi)\overleftrightarrow{\partial^2}(e^{(1-\rho)\partial^2}\phi) = \varphi\overleftrightarrow{e^{\partial^2}}\phi,$$

equation of motion and definition of field Φ, we have

$$\frac{d\mathcal{E}(t)}{dt} = \alpha'\partial\phi\partial^2\phi - \phi\partial\phi + \frac{\partial V}{\partial\Phi}\partial\Phi + \partial\Phi\overleftrightarrow{e^{k\partial^2}}(\alpha'\partial^2 - 1)e^{k\partial^2}\Phi \qquad (2.9)$$

$$= \partial\Phi\left[(-\alpha'\partial^2 + 1)e^{2k\partial^2}\Phi + \frac{\partial V}{\partial\Phi}\right] = 0. \quad \square$$

In the next paragraph we consider one important physical consequence of this energy conservation law.

2.2 Existence of the rolling solution

We already indicated that from physical perspective we are interested in solutions interpolating between two inequivalent vacua. We start our consideration by looking for stationary configurations Φ_0. Substituting it into equation of motion (2.3) we get $\Phi_0 = K^3\Phi_0^2$, which has two constant solutions: $\Phi_0 = 0$ and $\Phi_0 = K^{-3}$. We should thus be looking for solutions interpolating between those stationary points. The following claim though shows that energy conservation forbids existence of such solutions.

Claim 2. [4] *There do not exist continuous solutions of equation (2.3) which satisfy boundary conditions*

$$\lim \Phi(t) = \begin{cases} 0, & t \to \infty, \\ K^{-3}, & t \to -\infty \end{cases} \qquad (2.10)$$

or vice-versa (in terms $t \to -t$).

Proof. Let us assume existence of such solution and calculate energy at the extremum points, we get $E(\Phi = 0) = \Lambda$ and $E(\Phi = K^{-3}) = -\frac{1}{6}K^{-6} + \Lambda$, i.e. energy values at $t \to +\infty$ and $t \to -\infty$ are different what due to conservation law rules out existence of solutions satisfying (2.10). $\square$

3. The model coupled to the gravity

In this section we would like to consider tachyon dynamics in Friedmann-Robertson-Walker background what allows us to take into account gravity effects and makes the research more

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[4]Similar claim for p-adic string model was proved in [8], which rules out the possibility that tachyon may roll monotonically down from one extremum reaching the tachyon vacuum.

consistent from cosmological point of view. Consider the model

$$S = \frac{1}{g_0^2} \int d^4x \sqrt{-g} \left(\frac{m_p^2}{2} R + \frac{1}{2}\phi \Box \phi + \frac{1}{2}\phi^2 - \frac{1}{3}K^3\Phi^3 - \Lambda \right), \qquad (3.1)$$

here $m_p^2 = g_0^2 M_{pl}^2$ and we will work in units where $\alpha' = 1$. As a particular metric we will consider the FRW

$$ds^2 = -dt^2 + a^2(t)(dx_1^2 + dx_2^2 + dx_3^2),$$

for which the Beltrami-Laplace operator for spatially-homogeneous configurations takes the form $\Box = -\partial^2 - 3H(t)\partial = -\mathcal{D}_H^2$. Scalar field and Friedmann equations are

$$(-\mathcal{D}_H^2 + 1)e^{2k\mathcal{D}_H^2}\Phi = K^3\Phi^2, \qquad (3.2)$$

$$3H^2 = \frac{1}{m_p^2}\,\mathcal{E}, \qquad 3H^2 + 2\dot{H} = -\frac{1}{m_p^2}\,\mathcal{P}. \qquad (3.3)$$

Inclusion of the gravity considerably modifies the dynamics of the system. In Minkowski background we have shown that energy conservation law forbids dynamical interpolation between two inequivalent vacua. In FRW background energy of the scalar field alone does not conserve any more due to the Hubble term and as a result the restrictions on existence of such solutions no longer apply.

To find boundary conditions for possible solutions let us consider constant scalar field solution, Φ_0. In this case the scalar field equation (3.2) becomes

$$\Phi_0 = K^3\Phi_0^2, \qquad (3.4)$$

and first equation in (3.3)

$$3H_0^2 = \frac{1}{m_p^2}\,\mathcal{E}(\Phi_0). \qquad (3.5)$$

Equation (3.4) has two solutions: $\Phi_{0_1} = 0$ and $\Phi_{0_2} = K^{-3}$, substituting them into (3.5) we obtain corresponding values for a Hubble parameter $H_{0_1} = (18K^6)^{-1/2}$ and $H_{0_2} = 0$. Note that from cosmological perspective we are interested only in positive values for the Hubble function, so we can expect rolling solutions with the following boundary conditions

$$\lim \Phi(t) = \begin{cases} 0, & t \to \infty, \\ K^{-3}, & t \to -\infty, \end{cases} \qquad \lim H(t) = \begin{cases} (18K^6)^{-1/2}, & t \to \infty, \\ 0, & t \to -\infty, \end{cases} \qquad (3.6)$$

or vice-versa (in terms of $t \to -t$).

To analyze physical situation let us consider potential in which motion is expected. Naive extraction of potential from the model action (3.1) results in $V(\Phi) = -\frac{1}{2}\Phi^2 + \frac{1}{3}K^3\Phi^3 + \Lambda$. The constant Λ represents the D-brane tension which according to Sen's conjecture must be added to cancel negative energy appearing due to the presence of tachyon. We obtained two types of solutions. The first one is an ordinary rolling solution which starts from $\Phi = 0$ and goes towards configuration $\Phi = K^{-3}$ which is associated with the true vacuum. This solution can be interpreted as a description of the D-brane decay. The second one is a rolling solution which goes in the opposite direction, which
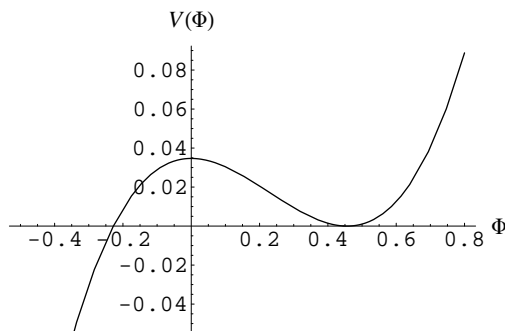
Figure 1: The potential.

appears in this model possibly because of the non-locality in the interaction. It is known that nonlocal dynamics has many interesting properties which are not possible in the local case. In particular the "slop effect" [8, 21, 27] which we can observe in the obtained solutions (figure 2, 3) when the scalar field goes beyond the values from which the scalar field configuration starts — situation which is not possible in the local models. Potentially a similar effect can initiate non-symmetry in the potential in ekpyrotic [39] and cyclic cosmology [40].

4. Numerical solution construction

In order to construct numerical solution we operate with scalar field equation of motion (3.2) and the difference of equations (3.3), specifically we solve

$$(-\mathcal{D}_{\mathrm{H}}^2 + 1)e^{2k\mathcal{D}_{\mathrm{H}}^2}\Phi = K^3\Phi^2, \tag{4.1a}$$

$$\dot{H} = -\frac{1}{2m_p^2}\left(\mathcal{P} + \mathcal{E}\right). \tag{4.1b}$$

We discretize equations (4.1a)–(4.1b) by introducing a lattice in t variable and then solving resulting system of nonlinear equations using iterative relaxation solver controlling error tolerance with discrete L_2 and L_∞ norms.

4.1 Discretization

When solving discretized equations (4.1a)–(4.1b) the nontrivial part from computational point of view is efficient evaluation of $e^{2k\rho\mathcal{D}_{\mathrm{H}}^2}\Phi$ for $\rho \in [0, 2]$. This operator could be interpreted in terms of initial value problem for the following diffusion equation with boundary conditions [31]

$$\partial_\rho\varphi(t, \rho) = 2k\left(\partial_t^2\varphi(t, \rho) + 3H(t)\partial_t\varphi(t, \rho)\right), \tag{4.2}$$

$$\varphi(0, t) = \Phi(t),$$

$$\varphi(\rho, \pm\infty) = \Phi(\pm\infty). \tag{4.3}$$

Once solution of this equation is constructed we have $e^{2k\rho\mathcal{D}_{\mathrm{H}}^2}\Phi(t) = \varphi(\rho, t)$.
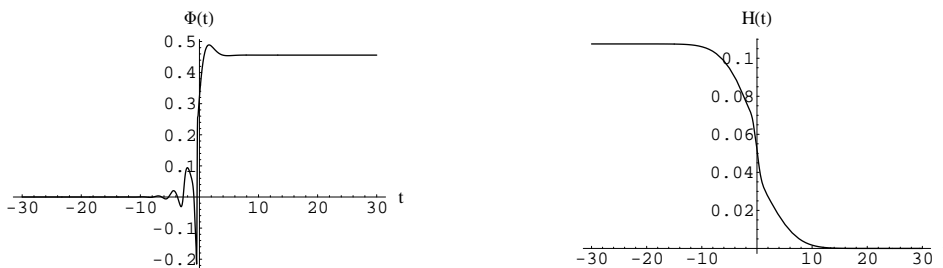
Figure 2: Solutions of the scalar field (3.2) and Friedmann equation (3.3) Φ and H (left to right) for $m_p^2 = 1$.

To solve (4.2) we used second order Crank-Nicholson scheme which is based on the following stepping procedure

$$\varphi(t, \rho + \Delta_\rho) = \left(1 + k\Delta_\rho \tilde{\mathcal{D}}_{\mathrm{H}}^2\right)\left(1 - k\Delta_\rho \tilde{\mathcal{D}}_{\mathrm{H}}^2\right)^{-1}\varphi + o(\Delta_\rho^2 \|\tilde{\mathcal{D}}_{\mathrm{H}}^2\|),$$

where $\tilde{\mathcal{D}}_{\mathrm{H}}^2$ denotes discretization of $\mathcal{D}_{\mathrm{H}}^2$ (it thus has a finite norm) and Δ_ρ is a step size along ρ variable. We used the following tri-diagonal discretization scheme for $\mathcal{D}_{\mathrm{H}}^2$

$$\tilde{\mathcal{D}}_{\mathrm{H}}^2 = \frac{1}{\Delta_t^2}\begin{pmatrix} -2 & 2 & 0 & \dots & 0 \\ 1 & -2 & 1 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 1 & -2 & 1 \\ 0 & \dots & 0 & -2 & 2 \end{pmatrix} + \frac{3}{2\Delta_t}\operatorname{diag}(H_{-N},\dots,H_N)\begin{pmatrix} 0 & 0 & 0 & \dots & 0 \\ -1 & 0 & 1 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & -1 & 0 & 1 \\ 0 & \dots & 0 & 0 & 0 \end{pmatrix},$$

where Δ_t is the step size along t and discretization of Hubble function is $H_k = H(k\Delta_t)$. This is a usual symmetric discretization scheme on the uniform lattice modified on the interface to guarantee boundary condition (4.1b) for smooth solutions which tend to constants as $t \to \pm\infty$.

4.2 Comparison of different methods

In order to exclude possible artifacts of the specific numerical scheme described above we tried Chebyshev-pseudospectral method which is known to generally have exponential convergence [41]. Such scheme is known to have very different properties [41] compared to finite difference scheme described above, but it produced the same results up to the approximation error which provides confidence in the existence of the rolling solutions reported in this work.

5. Rolling tachyon solution

Solutions of (3.2) and (3.3) are presented on figure 2, 3. It is interesting to note that on these figures shapes of scalar field look very similar up to reflection over vertical axis while shapes of Hubble function are different. In order to ensure that this is not an artefact or error in numerical calculations we performed the following test. First, we performed reflection,
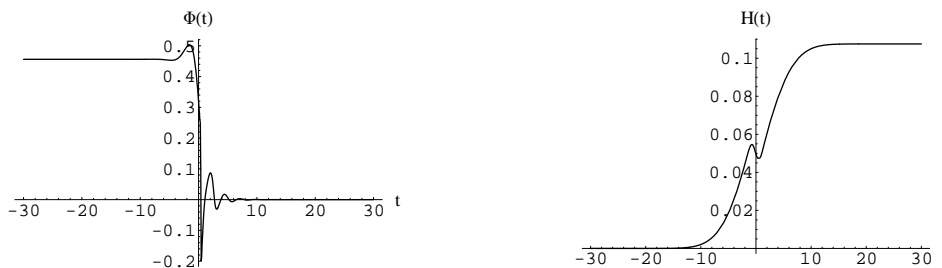
Figure 3: Solutions of the scalar field (3.2) and Friedmann equation (3.3) Φ and H (left to right) for $m_p^2 = 1$.

$t \to -t$, of the scalar field from figure 2 and compared it with scalar field from figure 3, while difference was small it was well above the error tolerance. We also numerically computed discrepancies between left and right sides of equations of motion where Hubble function was taken from one solution while scalar field was taken from another solution and mirrored. While the discrepancy for scalar field was small, the discrepancy for Hubble function was very large, larger than the Hubble function itself. This test makes us confident that qualitatively different shapes of Hubble function presented on figure 2, 3 are correct.

Another interesting property of the solution on figure 3 is that while scalar filed generally rolls down at first it climbs up. We already noted this "slop effect" in the end of section 3. In fact similar behavior is typical for solutions of nonlocal equations. In Minkowski metric such behavior was noted by many authors who used different numerical techniques [21, 27, 42]. It is interesting that in FRW metric this property is preserved.

5.1 Solutions for different m_p

Motivated by the fact that string scale does not exactly coincide with Planck mass and as a consequence there is some freedom in settling m_p^2 we investigated how the shape of solutions behaves for different m_p^2. Solutions of (3.2), (3.3) for different values of m_p^2 are presented on figure 4 and 5.

We can see that the profiles for different m_p^2 for the first case are very similar. It is interesting to note how on figure 5 with the growth of m_p^2 oscillation in the profile of Hubble function disappears.

5.2 Particular case of p-adic action

The p-adic string model represents a popular toy-model that was proposed in [34]. The model contains one scalar field and nonlocal interaction. Formulated in arbitrary space-time dimensionality the model has a parameter p, which is initially taken to be a prime number. It was shown later that the model describes quite well some physical processes in string theory, despite its intrinsic limitations. Recently this model started to attract attention as a cosmological toy model for describing inflation [35, 36, 43]. The p-adic string model was later considered as being minimally coupled to gravity in Friedmann-Robertson-Walker metric. In this subsection we would like to show some intriguing properties within this context.
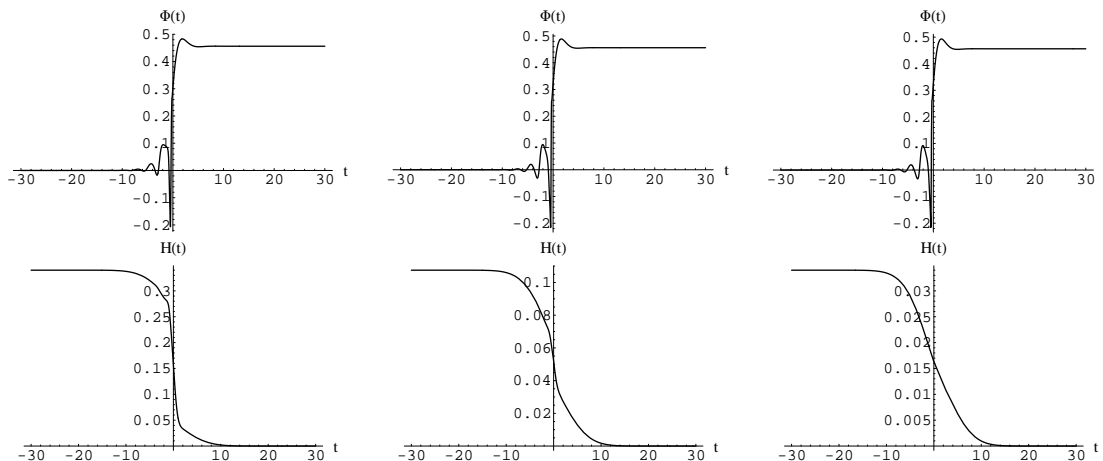
Figure 4: Solutions of the scalar field (3.2) and Friedmann equation (3.3) Φ, H for $m_p^2 = 0.1, 1, 10$ (left to right).
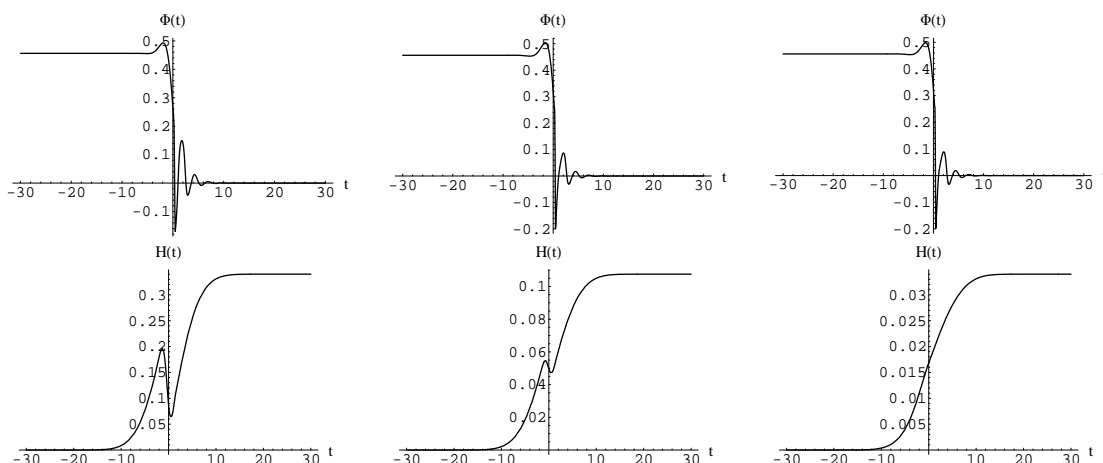
Figure 5: Solutions of the scalar field (3.2) and Friedmann equation (3.3) Φ, H for $m_p^2 = 0.1, 1, 10$ (left to right).

The p-adic action is given by

$$S_p = \int d^d x L_p = \frac{1}{g_p^2} \int d^d x \left[-\frac{1}{2} \phi p^{-\frac{1}{2}\Box} \phi + \frac{1}{p+1} \phi^{p+1} \right], \quad \frac{1}{g_p^2} = \frac{1}{g^2} \frac{p^2}{p-1}. \tag{5.1}$$

The infinite number of space-time derivatives are manifest in the pseudo-differential operator $p^{-\frac{1}{2}\Box}$, where $\Box = -\partial^2 + \nabla^2$, $p^{-\frac{1}{2}\Box} = e^{-\frac{1}{2}\ln p\Box}$.

Considering action (5.1) for $p = 2$ after the field redefinition $\varphi = e^{-\frac{1}{4}\ln p\Box} \phi$ we have

$$S_2 = \int d^d x L_2 = \frac{1}{g_p^2} \int d^d x \left[-\frac{1}{2} \varphi^2 + \frac{1}{3} (e^{\frac{\ln 2}{4}\Box} \varphi)^3 \right], \tag{5.2}$$

which looks rather similar to the tachyon OSFT action in the level truncation approximation without kinetic term and with only other difference in common signs.

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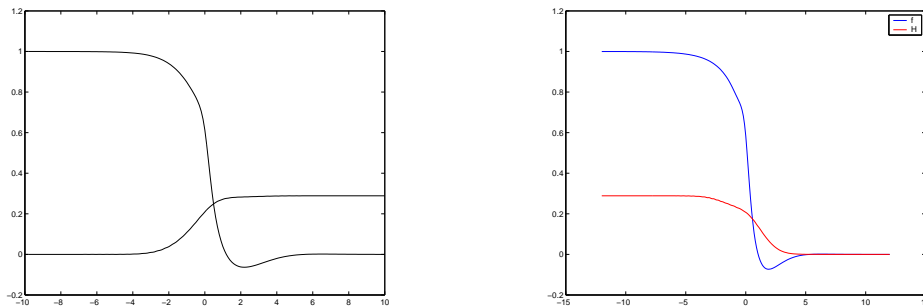
Figure 6: Solutions of the Friedmann equations Φ, H: negative sign in front of the Lagrangian (left), positive sign in front of the Lagrangian (right).
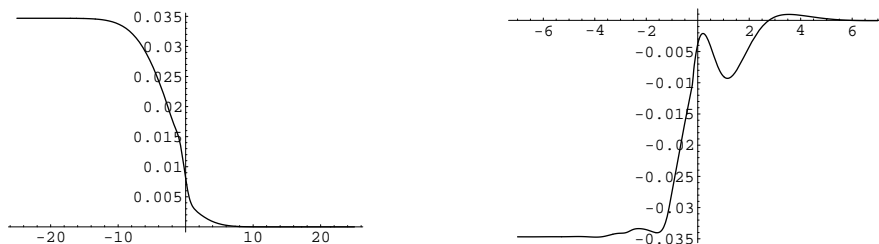
Figure 7: Energy (left) and pressure (right) for solutions from figure 2.

Indeed, if we denote the approximate lagrangian for the OSFT case

$$L_{OSFT_{\text{approx}}} = \frac{1}{2}\phi^2 - \frac{1}{3}K^3(e^{k\Box}\phi)^3$$

we see that for the case of p-adic string model with $p = 2$ if we neglect the difference in the factors in exponential operator we have

$$L_{p=2} = -L_{OSFT_{\text{approx}}}.$$

This observation does not affect dynamic in the case of Minkowski space-time because sign in front of the Lagrangian does not enter scalar field equation. The situation changes significantly in Friedmann-Robertson-Walker background because the sign affects equation for Hubble function. Thus if in the case of OSFT action in approximation neglecting kinetic term we had monotonically increasing Hubble function in the case of p-adic model for $p = 2$ we obtain monotonically decreasing Hubble function for the same scalar field configuration. Corresponding solutions are presented on figure 6.

One of the issues is that the p-adic sting model in the FRW case is not commonly established and choosing the right sign is nontrivial, especially due to the absence of usual canonical kinetic term.

5.3 Energy and Pressure for Rolling Tachyon Solutions

Usually when considering dynamics in nontrivial background such as FRW it is instructive to consider simpler case of Minkowski metric. The intriguing fact though is that there is no such a possibility since energy conservation law forbids in Minkowksi case solutions like
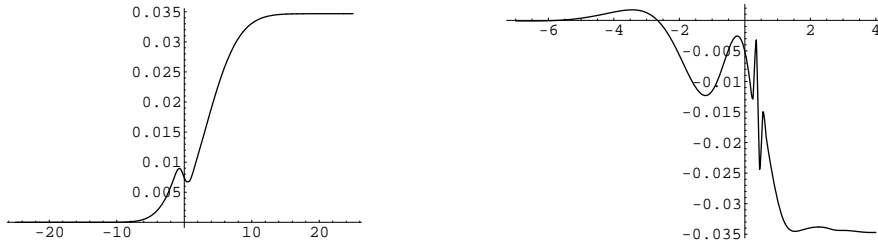
Figure 8: Energy (left) and pressure (right) for solutions from figure 3.

those presented in figure 2, 3. Nevertheless it is possible to qualitatively characterize new features of the system.

Dynamics of the energy and pressure (2.5) for the scalar field from figure 2 is presented on figure 7. We can see that dynamics is different from what one might expect in Minkowski case. We obtained solution for which both energy and pressure nontrivially tend to zero at large times. For the solution from figure 3 the energy and pressure dynamics is different, see figure 8. The pressure starts from zero and goes to a negative constant, while the energy starts from zero and goes to a constant of the same value with positive sign.

In both cases we obtained nontrivial partially negative pressure and like in [3] non-trivial equation of state what might be especially interesting in the light of cosmological applications.

5.4 Cosmological applications

To discuss cosmological consequences it is interesting to refer to the concluding remarks and open questions of the work [8] which in fact initiated a whole series of investigation of nonlocal dynamics in the models with infinitely many time derivatives. The paper [8] considered string field theory and p-adic theory in Minkowski space time. It was noted that the most puzzling result from physical point of view is that none of the solutions obtained there appeared to represent tachyon matter. In other words solutions with varying values of energy for which pressure tends to zero at late times were not found. It is notable that solutions obtained in this work have such behavior. As we can see on figure 7 the pressure goes to zero for long times at the end of the evolution. In fact this represents first alternative of possible cosmological applications of the obtained solutions.

The second alternative is to consider obtained solutions in the context of description of very early Universe. This investigation is motivated by the shape of the solutions with decreasing positive Hubble function presented at the figure 2. The research in this direction was started in [35] based on p-adic string model and followed by [43] in which CSSFT string model was considered as well.

The third alternative is to consider configuration from figure 3 in the context of late time acceleration.

6. Summary

In this paper we have studied dynamics with infinitely many time derivatives in Friedmann-Robertson-Walker background. The main results are:

- It was shown that field configurations which interpolate between vacua with different scalar field energies are possible in the FRW background and corresponding rolling solutions were obtained.

- We found nontrivial dynamics in the FRW background which differs from dynamics in Minkowski case. Particularly, we found interpolating solution which goes from maximum to the true minimum for which the pressure approaches zero at long times while energy varies starting from the constant which is equal to the brane tension and tends to zero at the end of the evolution.

Among main results we have also considered in details the properties of constructed solutions and have shown that the evolution is possible in both directions, and what is even more puzzling profiles for Hubble functions are different in those cases. In one direction Hubble function is an increasing almost monotonic function, while in another we observe significant oscillations during evolution. Moreover it was shown that by varying string scale we can change the shape of Hubble function and for some string/Planck scale ratio the oscillations disappear. We also considered dynamics in p-adic string model for particular value of a parameter $p = 2$ and have shown that dynamics in the FRW background drastically differs from the corresponding one in usual string theory.

It is interesting to note that for numerical construction of solutions presented here we had to abandon explicit iterative techniques which were very successful in previous investigations of rolling tachyon solutions, see [31, 43] and references therein. Instead we used slower but more generic relaxation methods with Crank-Nicholson scheme to compute nonlocal operators.

Acknowledgments

A. Useful identity

In this appendix we will prove the following identity

$$\int\limits_0^1 d\rho (e^{\rho\partial^2}\varphi)\overleftrightarrow{\partial^2}(e^{(1-\rho)\partial^2}\phi) = \varphi e^{\overleftrightarrow{\partial^2}}\phi, \tag{A.1}$$

where symbol $e^{\rho\partial^2}\varphi$ is defined in (2.7).

It is well known, that for functions $\Phi(t)$ which are continuous and bounded on the real axis the following identity have a place

$$\lim_{\rho \to +0} C_\rho[\Phi](t) = \Phi(t) \tag{A.2}$$

We can formulate the following lemma

Lemma 1. *For continuous and bounded functions $\psi(t)$ and $\varphi(t)$ the following identity takes a place*

$$\int\limits_0^1 d\rho\, C_\rho[\varphi](t) \overleftrightarrow{\partial^2} C_{1-\rho}[\psi](t) = \varphi \overleftrightarrow{C_1} \psi, \quad \partial^2 \equiv \frac{d^2}{dt^2} \tag{A.3}$$

where left side is understood as

$$\lim_{\epsilon_1 \to +0} \lim_{\epsilon_2 \to +0} \int_{\epsilon_1}^{1-\epsilon_2} d\rho d\rho\, C_\rho[\varphi](t) \overleftrightarrow{\partial^2} C_{1-\rho}[\psi](t)$$

and right hand side as

$$\varphi \overleftrightarrow{C_1} \psi = \varphi(t) C_1[\psi](t) - C_1[\varphi](t)\psi(t)$$

Proof. We have

$$\int\limits_{\varepsilon_1}^{1-\varepsilon_2} d\rho\, C_\rho[\varphi] \overleftrightarrow{\partial^2} C_{1-\rho}[\psi] = \int\limits_{\varepsilon_1}^{1-\varepsilon_2} d\rho\, C_\rho[\varphi](\partial^2 C_{1-\rho}[\psi]) - \int\limits_{\varepsilon_1}^{1-\varepsilon_2} d\rho(\partial^2 C_\rho[\varphi]) 2 C_{1-\rho}[\psi].$$

We will use the fact that for $\rho > 0$ the function $C_\rho[\varphi](t)$ is a solution of the diffusion equation, i.e

$$\frac{\partial^2}{\partial t^2} C_\rho[\varphi](t) = \frac{\partial}{\partial \rho} C_\rho[\varphi](t), \qquad \rho > 0 \tag{A.4}$$

$$\frac{\partial^2}{\partial t^2} C_{1-\rho}[\varphi](t) = -\frac{\partial}{\partial \rho} C_{1-\rho}[\varphi](t), \qquad \rho > 0 \tag{A.5}$$

The proofs of these identities follow from integral representation.

Taking into account the identities written above we get

$$\int\limits_{\varepsilon_1}^{1-\varepsilon_2} d\rho\, C_\rho[\varphi] \overleftrightarrow{\partial^2} C_{1-\rho}[\psi] = \int\limits_{\varepsilon_1}^{1-\varepsilon_2} d\rho\, C_\rho[\varphi](\partial^2 C_{1-\rho}[\psi]) - \int\limits_{\varepsilon_1}^{1-\varepsilon_2} d\rho(\partial^2 C_\rho[\varphi]) C_{1-\rho}[\psi] \tag{A.6}$$

$$= -\int\limits_{\varepsilon_1}^{1-\varepsilon_2} d\rho \left(C_\rho[\varphi] \frac{\partial}{\partial \rho} C_{1-\rho}[\psi] + \frac{\partial}{\partial \rho} C_\rho[\varphi] C_{1-\rho}[\psi] \right)$$

$$= -\int\limits_{\varepsilon_1}^{1-\varepsilon_2} d\rho \frac{\partial}{\partial \rho} (C_\rho[\varphi] C_{1-\rho}[\psi])$$

$$= -C_{1-\varepsilon_2}[\varphi] C_{\varepsilon_2}[\psi] + C_{\varepsilon_1}[\varphi] C_{1-\varepsilon_1}[\psi]$$

and (A.2), we can take the limit $\varepsilon_1 \to +0$, $\varepsilon_2 \to +0$ and get (A.3). $\qquad \square$

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